

NOTATION

μ , σ , ρ , dynamic viscosity, surface tension, and the density of the liquid, respectively. The quantity $Re = \rho W d / \mu$ is the Reynold's number, W is the jet's speed, and $Lp = \rho \sigma d_j / \mu^2$ is Laplace's number.

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INDUCED CAPILLARY JET BREAKDOWN

N. A. Razumovskii

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A model is proposed for the induced capillary breakdown of a liquid jet with small surface tension and viscosity, on the basis of numerical solution of the quasi-one-dimensional equations of jet flow. The algorithm employed permits the description of flow from the source to the point of breakdown.

The mathematical modeling of capillary jet breakdown has a rich history, beginning with studies of the capillary instability of a liquid cylinder [1, 2]. Induced breakdown taking account of the mechanism of jet excitation was first described in [3, 4]. Two basic trends in the mathematical modeling of induced jet breakdown have their origin in [4]. The first, based on perturbation theory, was developed in [5-13]. Common to these studies is the solution of hydrodynamic equations in the form of asymptotic series in which 2-3 terms are retained. As a rule, the relative change in jet radius is used as the small parameter. Formulas are obtained for the jet length from the source to the point of breakdown, a qualitative description of the process of satellite formation is proposed [8, 10], and the satellite masses are calculated [9]. The basic deficiency of such work is that the region of applicability of the asymptotic solution is limited to the initial section of the jet, where the perturbation is sufficiently small. In fact, the series may be regarded as asymptotic if the subsequent terms are small corrections to the first terms. Otherwise, it is impossible to discard all but the first few terms. Close to the point of breakdown, none of the initial approximations employed is close to the accurate solution. Therefore, the most interesting processes of drop formation and satellite formation characterized by a large perturbation amplitude are only qualitatively described.

The other trend, which is based on numerical methods, is represented by [14-16]. The numerical solution of the complete hydrodynamic equations in a jet is fairly complex and requires considerable machine time. In [15], such calculations were undertaken for the case of very large flow rates (Weber number $We \sim 10,000$), when the surface tension is insignificant. This simplifies the calculation, but necessitates taking complete account of the dependence of the velocity and pressure on the radial coordinate. Calculations show that, for a jet at moderate velocity ($We \sim 100-500$), in a number of cases, quasi-one-dimensional de-

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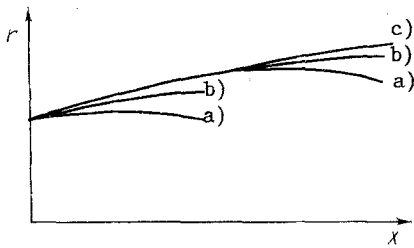


Fig. 1

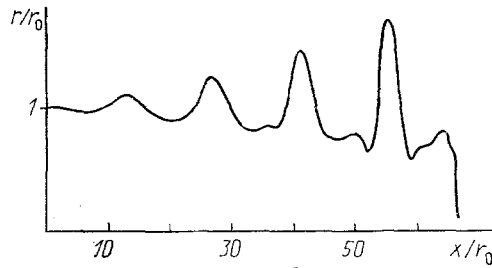


Fig. 2

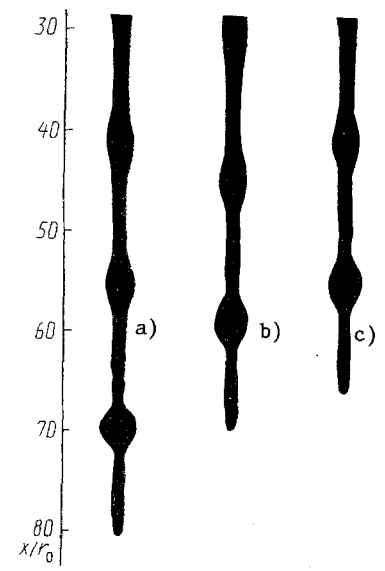


Fig. 3

Fig. 1. Scheme of numerical solution of quasi-one-dimensional equations describing capillary jet breakdown (x is the coordinate measured from the source to the jet axis): a, b) first and second approximations to accurate solution (c).

Fig. 2. Dependence of the jet radius r on the axial coordinate x . The flow is characterized by Reynolds number $Re = 1000$ and Weber number $We = 200$. The ratio of the jet radius at the source to the spatial "period" of the perturbation λ is $r_0/\lambda = 0.07$. The initial flow velocity pulsates with an amplitude of 0.03 of the mean value.

Fig. 3. Theoretical jet profiles with different liquid viscosities. For all the jets, $We = 200$, $r_0/\lambda = 0.07$, and the amplitude of the initial-velocity pulsations is 0.03 of the mean value: a) $Re = 30$; b) 100; c) 1000.

scription of the flow is possible, approximating the radial dependence of the velocity and pressure by expressions with finite parameters. For example, in the simplest approximation of small water droplets, it is assumed that the pressure and axial velocity component are constant over the jet cross section.

In [14, 16], evolutionary problems were formulated and solved, since in these problems it is simpler to eliminate numerical instability. The formulation of the evolutionary problem includes the description of the initial flow and the boundary conditions at the source and the point of jet breakdown. In view of the indeterminacy of the initial and boundary conditions at the breakdown point, an inadequate but related model problem was considered in [14]: the breakdown of an infinite liquid cylinder subjected to a small initial perturbation. This method was introduced in [1]. The solution obtained permits the prediction of the time of breakdown and hence the jet length. At the same time, it cannot be used in investigating the conditions of satellite formation, since it shows the presence of a satellite in all conditions. For more accurate description of satellite formation, the spatial aperiodicity of the jet associated with the increase in the perturbation on moving away from the source must be taken into account. To this end, the evolution of the flow of a semiinfinite jet with initially constant velocity was considered in [16]. The flow velocity subsequently begins to pulsate in time. The perturbation gradually fills the initial section of the jet, increasing to some maximum on moving away from the source, and then rapidly falling. The position of the maximum moves at approximately the velocity of the jet, and is determined by the time from the instant of switching on the velocity pulsations of the jet. The perturbation in the initial section of the jet may be regarded as steady with some degree of accuracy. With the same accuracy, it may be assumed that the section of jet further from the source than the points of observation has no influence on the flow at the given point. Therefore, where steady oscillations are established in the flow, the solution obtained corresponds to the flow of the decomposing jet. Calculations show that this is the case over almost the whole jet, excluding the last few periods close to the traveling maximum. The calculation ends when the jet radius at any point becomes zero. Thus, in [16], a mathematical model of a decomposing jet which is adequate everywhere except for the last few periods before the point

of breakdown has been constructed. The other deficiency is the large volume of computations required.

In the present work, a numerical method applicable up to the point of breakdown is proposed; this model makes no unjustified assumptions and does not require a large volume of calculations. It describes a liquid jet with a small surface tension and viscosity. The time-periodic flow of a free jet corresponding to a radius and flow velocity specified at the source and expressed by periodic functions of the time is sought. The flow is described by quasi-one-dimensional equations, for example, the equations of fine water droplets. In the general case, these data are inadequate for unique determination of the solution. Uniqueness appears on taking account of the relations $Re \gg 1$, $We \gg 1$, which express the smallness of the viscosity and surface tension, and requiring that the derivatives of the solution be finite. This is easily understood for the example of solving a linearized equation. As is known [17], this equation has four linearly independent solutions when $We \gg 1$ and $Re \gg 1$: two long waves of finely disperse water and two short capillary waves. The requirement that the derivatives of the solution be finite means that the amplitudes of the short capillary waves are neglected. The two boundary conditions given are sufficient for unique determination of the amplitudes of the long waves. A necessary condition of finiteness of the derivatives is to limit the frequency spectrum of the solution to frequencies of finite multiplicity. Essentially, these constraints have already been imposed in deriving the quasi-one-dimensional equations of flow. In [5-13], they are present in even stronger form: harmonics of no more than third order are taken into account. In the model proposed here, the number of harmonics taken into account is determined by the flow parameters and falls in the range 5-10.

The scheme for application of the algorithm is shown in Fig. 1, and is as follows. The jet is divided into segments. The jet radius and flow velocity are determined as a function of the time successively in these segments, moving away from the source. In each segment, the finite-difference solution is constructed by the successive-approximation method. The first approximation corresponds to flow with no viscosity and no surface tension. Each subsequent approximation contains corrections taking account of the viscosity and surface tension calculated on the basis of the previous approximation. The segments are chosen to be short enough that the corrections remain small within each one. The given assumptions cease to be valid in the immediate vicinity of the breakdown point.

The algorithm proposed is based on the JET program in Pascal. Calculations on an IZOT-1016 computer require approximately 10-min machine time per jet. Examples of the theoretical profiles are shown in Figs. 2 and 3. They correspond to the following initial conditions. The jet radius at the source is constant (r_0), and the velocity v depends on the time t as follows

$$v = v_0 [1 + 0,03 \sin(0,4v_0 t/r_0)],$$

where $v_0 = \text{const}$. The Weber number is 200. The graph corresponding to $Re = 1000$ (Fig. 2) shows that the initial section of the jet is satisfactorily described by the linearized equations. As shown in [17], there is a section of linear increase in the perturbation close to the source, which transforms to a section of exponential growth. Jet profiles corresponding to different values of the viscosity are shown in Fig. 3. It is evident that, at large viscosity, jet breakdown is preceded by the stage of "beads on a thread," which is known from experiments and was predicted theoretically in [18] on the basis of an asymptotic approach.

NOTATION

We , Weber number; Re , Reynolds number; r , jet radius; r_0 , jet radius at source; v , flow velocity; v_0 , constant component of flow velocity; t , time; λ , spatial "period" of perturbation.

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POSSIBLE APPLICATIONS OF FIBER-OPTIC METHODS OF MEASURING PHYSICAL
QUANTITIES FOR DIAGNOSTICS OF THE BREAKUP OF FREE JETS

V. V. Osipov and V. P. Ogorodnikov

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The possible use of fiber-optic sensors of physical quantities to measure the parameters of drop generators is examined. The proposed fiber-optic devices can be used successfully in experimental studies of the monodisperse breakup of free jets.

Free liquid jets are used extensively in modern engineering and technology, e.g., in cryodisperse [1] and cryochemical technologies [2]. This stimulates the development of experimental methods of studying free jets. Measurements of the velocity profiles in thin jets (less than 1 mm in diameter) is the most complicated. Here we consider the possible use of fiber-optic methods to study the characteristics of jets and the monodisperse macro-particles formed as a result of induced capillary breakup of streams.

The restructuring of the velocity profile in a jet can be studied, e.g., with a Pitot tube, but its diameter should be 0.05-0.01 of the jet diameter d . If $d = 500 \mu\text{m}$, for example, the diameter of the Pitot-tube head should be 5-10 μm . Heads of this diameter are technically feasible but are very difficult to use because a sensor of this diameter has a very high inertia if the dynamic pressure on it is measured with an ordinary U-tube manometer. As our estimate showed [3] if the Pitot-tube diameter $d_p = 5 \cdot 10^{-6} \text{ m}$, the length of the Pitot-tube head $l = 5 \cdot 10^{-4} \text{ m}$, the liquid studied is water with density 10^3 kg/m^3 and dynamic viscosity $10^{-3} \text{ N}\cdot\text{sec/m}^2$, and if the manometer tube diameter $d_m = 10^{-2} \text{ m}$, the time constant of such a sensor is $2 \cdot 10^8 \text{ sec}$, which is unacceptable. The time constant of such a sensor must be reduced by roughly six orders of magnitude if the sensor is to be functional.

For this purpose we propose a fiber-optic manometer with a magnetohydrodynamic pump as reverse transducer, whose diagram is shown in Fig. 1. The manometer consists of a round U-shaped tube 1, whose lower segment is made in the form of a flat channel with copper buses 2 along the smaller sides and is placed in the gap of a permanent magnet 3. The voltage to the copper buses is supplied by a regulated source 10, which is controlled by a microcomputer.

A sharp boundary between the wetted and unwetted surfaces is organized in the tubes of the manometer to eliminate the effect of surface tension on the manometer readings. For this purpose a wetted ring 4 (e.g., of amalgamated copper) is inserted into the unwetted tubes of the manometer. If the volume of mercury in the manometer is such that the interface is at the level of the boundary between the wetted and unwetted surfaces of the tube, the interface is flat and the role of the surface tension is minimized. The change in the mercury

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